Enrollment No: ______________________ Exam Seat No:______________________

C.U.SHAH UNIVERSITY

Summer Examination-2017

Subject Name : Introduction to Quantum Mechanics

Subject Code : 4SC06QMC1 Branch: B.Sc. (Physics)

Semester : 6 Date :11/04/2017 Time : 02:30 To 05:30 Marks : 70

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 Attempt the following questions: (14)

a) Define the 'Matter Waves' represented by the wave function in quantum mechanics. (01)

b) What is D'Broglie's hypothesis? (01)

c) Write formulas for (01)
(1) Energy **E** =…….according to Plank's theory, and
(2) Momentum **P** =…… according to D' Brogly's theory.

d) Define 'Wave Packet' in quantum mechanics. (01)

e) Define Hamiltonian operator $\boldsymbol{H}$. Write its formula. (01)

f) What do you mean by $*$ in the formula $|\Psi|^2 = \Psi\Psi*$. Name $\Psi *$ in $|\Psi|^2 = \Psi\Psi*$ (01)

g) Name the process used for normalization of " a (non-normalized) wave function with infinite norm"? (01)

h) Write the time independent Schrödinger equation. (01)

i) Define : Stationary states. (01)

j) Write Schrödinger Equation for the system of n particles. (01)

k) Define : Dirac delta function. (01)

l) Define : Self Adjoint Operator. (01)

m) Define : 'Conversation of Probability' in terms of Quantum Mechanics. (01)

n) For a self adjoint operator, what is implied by conditions : (01)
$\langle A \rangle = \langle A^* \rangle$ and $a = a^*$

Attempt any Four questions from question No.-2 to question No.-8

Q-2 **Attempt all questions** **(14)**

A Derive Schrödinger Equation for free particle in one dimension; Obtain three dimensional generalized form of Schrödinger Equation. (07)

B Obtain Ehrenfest Theorem by deriving necessary formulas for (07)
(1) Expectation value of moment and also
(2) Expectation Value in terms of Newton's second law of motion that proves expectation value of moment is $m.\frac{d<x>}{dt} = < P_x >$

Q-3 **Attempt all questions** **(14)**

A Give statements of the fundamental postulates of wave mechanics. Prove any one. (07)

B Define : Adjoint of an operator. (07)
Derive the following properties of *Adjoint Operators.*

(i) $(A + B)^+ = A^+ + B^+$

(ii) $(A.B)^+ = B^+ A^+$

(iii) $C.A^+ = C^* A^+$

(iv) $(A^+)^+ = A$

Q-4 **Attempt all questions** **(14)**

A Discuss in detail "the state with minimum value for uncertainty product" in which prove that the uncertainty product of x and P_x is minimum as the probability of finding the particle is maximum. (07)

B Discuss the Schrödinger equation and energy Eigen values in terms of the ' simple harmonic oscillator'. (07)

Q-5 **Attempt all questions** **(14)**

A Obtain the Normalized Wave Function of the following each. (08)

(i) $\Psi = a.exp\ i(kx - wt)$ where $-1 \le x \le 1$

(ii) $\chi = exp\ (-i\ \theta)$ where $0 < \theta < 2\pi$

(iii) $\Psi'(\phi) = A\sin(mn\phi)$ where $0 < \phi < 2\pi$

(iv) $\phi = A\sin\phi$ where $0 < \phi < \pi$

B Normalize the *Wave Function* $\Psi' = \propto . e^{\pm i(kx - wt)}$ with infinite norms by the box normalization method. Where $-L \le x \le L$. (04)

C Show that the commutator of the position and momentum do not vanishes for a particle; as $[x, P_x] = [y, P_y] = [z, P_z] = i\hbar$ (02)

Q-6 **Attempt all questions** **(14)**

A Find the Eigen values of the following Eigen functions for operator A = d^2/dx^2 (08)

(i) $\Psi\ (x) = sin\ 2x$ **(iii)** $\phi\ (x) = e^{(nx)}$

(ii) χ (x) = cos x **(iv)** Φ (x) = $sin^2 x$

B Prove the following : (06)

(i) $[P_x, x] = [P_y, y] = [P_z, z] = -i\hbar$

(ii) $[y, P_x] = [z, P_y] = [x, P_z] = 0$

Q-7 **Attempt all questions** **(14)**

A Prove that : (03)

(i) $[x, P_x^n] = n\, i\, \hbar\, P_x^{n-1}$

(ii) $[y, P_y^n] = n\, i\, \hbar\, P_y^{n-1}$

(iii) $[z, P_z^n] = n\, i\, \hbar\, P_z^{n-1}$

B For angular momentum prove that: (04)

(i) $[L_x, L_y] = n\, i\, \hbar\, L_z$

(ii) $[L_y, L_z] = n\, i\, \hbar\, L_x$

(iii) $[L_z, L_x] = n\, i\, \hbar\, L_y$

C Angular momentum of a particle with position vector $\vec{r}$ and linear momentum $\vec{p}$ is $\vec{r} \times \vec{p}$. Obtain the operators for components L_x, L_y and L_z of the angular momentum. (04)

D For given wave function $\Psi = \sqrt{\frac{2}{L}} . sin\left(\frac{n\pi x}{L}\right)$, (03)

calculate the Expectation Value of x for a particle in a box having the volume L^3

Q-8 **Attempt all questions** **(14)**

A Prove that ($A^{\dagger}A$) is a self adjoint. (02)

B If A & B are self adjoint and commute each other than prove that AB is also self adjoint. (03)

C Obtain $[p,\ f(x)]$ where $f(x)$ is some function of operator x . (03)

D Given that $x = i\hbar \frac{\partial}{\partial p}$, obtain $[x,\ f(p)]$ (03)

E The wave function of a particle is $\left(1/\sqrt{\pi}\right)^{-1/2} e^{-x^2}$. (03)

Find the expectation value of its linear momentum.